

# FACULTY OF MECHANICAL ENGINEERING, CTU IN PRAGUE

## MATHEMATICS ENTRANCE EXAMINATION

bachelor studies, year 2023, version B1

Surname		First name	
Birth date		Registration number	
Corrected by		Points	

The time for the examination is 60 minutes, no aids are allowed except writing aids. Perform the necessary auxiliary calculations on the attached blank papers. Write the result distinctly and unambiguously into the thick framed box next to the task. A maximum of 2 points can be obtained for each task.

Exercise	Task	Result	Points
1.	Determine the range $D(f)$ of function $f(x) = \sqrt{\frac{x+2}{4x-6}}$ . Write the result as interval(s).	$(-\infty; -2) \cup (\frac{3}{2}; \infty)$	
2.	Solve the equation $x(a-1) + a(x+4) = 2$ with unknown $x \in \mathbb{R}$ and parameter $a \in \mathbb{R}$ .	$a = \frac{1}{2} \Rightarrow x \in \mathbb{R}; a \neq \frac{1}{2} \Rightarrow x = -2$	
3.	Write down the equation of quadratic function, that has a graph passing through points $K = [0; -3]$ , $L = [1; 0]$ and $M = [-1; -4]$ .	$f(x) = x^2 + 2x - 3$	
4.	Solve the equation $3\sqrt{x+5} - 5 = x$ in the system of real numbers $\mathbb{R}$ .	$x \in \{-5; 4\}$	
5.	Solve the inequality $ x - \sqrt{3}  > 2 + 5\sqrt{3}$ in the system of real numbers $\mathbb{R}$ .	$(-\infty; -2 - 4\sqrt{3}) \cup (2 + 6\sqrt{3}; \infty)$	
6.	Solve the inequality $\frac{2x+3}{x-1} < 1$ in the system of real numbers $\mathbb{R}$ .	$(-4; 1)$	
7.	Determine all solutions of equation $2\cos^2(x) - \cos(x) - 1 = 0$ for $x \in \langle 0; 2\pi \rangle$ .	$x \in \{0; \frac{2}{3}\pi; \frac{4}{3}\pi; 2\pi\}$	
8.	Solve the equation $3^{5-x^2} = 3^{x+3}$ in the system of real numbers $\mathbb{R}$ .	$x \in \{1; -2\}$	
9.	Determine the magnitude of the complex number $z = \frac{\sqrt{3}+1}{3} - \frac{\sqrt{3}-1}{3}i$ .	$ z  = \frac{2}{3}\sqrt{2}$	
10.	Solve the equation $\log_2(x+1) = 3$ in the system of real numbers $\mathbb{R}$ .	$x = 7$	
11.	Consider the arithmetic sequence with $a_4 = 9$ and $a_{10} = 21$ . Determine the difference $d$ and the first term $a_1$ .	$a_1 = 3, d = 2$	
12.	Determine the position of two straight lines $p: x + 2y - 1 = 0$ and $q: 2x - y + 4 = 0$ .	Straight lines are intersecting	

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Faculty of Mechanical Engineering, CTU in Prague  
Entrance examination for bachelor studies, 2019

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Mathematics examination example

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**Surname:**

First Name:

Date of birth:

**Registration number:**

Corrected by:

Points:

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Perform the necessary auxiliary calculations on the attached blank papers. Write the result obtained by calculation in the right column next to the task assignment. The result must be written clearly and unambiguously, otherwise it will not be accepted. For the correct answer you receive two points for each task. If the result has two parts, then one point can be obtained for each of them. After finishing the examination, submit the completed paper with the assignment and all papers with auxiliary calculations.

The time allotted for examination is 60 minutes. No aids are allowed other than the information printed on the examination paper (i.e. not a calculator or mathematical tables).

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1. Determine the range  $D(f)$  of function  $f$  :

$y = \frac{x}{\sqrt{2x^2 + 3x - 5}}$ . Write the result as interval(s).

$$x \in \left(-\infty, -\frac{5}{2}\right) \cup (1, \infty)$$

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2. For which values of variable  $p$  is the expression  $\left(3 - \frac{4p+1}{p+3}\right)$  equal to zero?

$$p = 8$$

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3. Determine the coordinates  $x_0, y_0$  of the vertex of the parabola which is a graph of the function  $y = x^2 - 4x + 2$ .

$$x_0 = 2, y_0 = -2$$

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4. Solve equation  $\frac{2}{\sqrt{4x-2}} - 1 = 0$  in the system of real numbers  $\mathbb{R}$ .

$$x = \frac{3}{2}$$

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5. Solve equation  $(x+2)(4x-2) = 3x-3$  in the system of real numbers  $\mathbb{R}$ .

$$x_1 = \frac{1}{4}, x_2 = -1$$

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6. Solve inequality  $\left|x - \frac{7}{2}\right| \leq \frac{5}{2}$  for  $x \in \mathbb{R}$ .

$$x \in [1, 6]$$

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7. Determine all solutions of equation:  $\sin^2 x + 2 \sin x = 0$ .

$$x = 0 + k\pi, k \in \mathbb{Z}$$

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8. Determine the expression  $V$  if

$$\ln V = 3 \ln(2-x) - \frac{1}{2} \ln(x+7).$$

Determine all  $x \in \mathbb{R}$  for which  $\ln V$  is defined.

$$V = \frac{(2-x)^3}{\sqrt{x+7}}, x \in (-7, 2)$$

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9. Write the complex number  $z = (3-3i)(5-2i) - (6-17i)$  in a form  $a+bi$ . Determine the magnitude of this complex number.

$$z = 3-4i, \|z\| = 5$$

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10. Solve equation  $2^{3-2x}2^{3x-1} = 1$  in the system of real numbers  $\mathbb{R}$ .

$$x = -2$$

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11. Consider the arithmetic sequence with  $a_3 = 5$  and sum  $s_6 = 39$ . Determine the term  $a_1$  and the difference  $d$ .

$$a_1 = -1, d = 3$$

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12. Two straight lines  $p : 3x + 3y = \frac{3}{4}$  and  $q : 2x - y = -\frac{5}{2}$  are given. Determine the coordinate of intersection point  $P = [x, y]$  of these straight lines.

$$P = \left[-\frac{3}{4}, 1\right]$$

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